How Does the Market-Based Intermediary Sector Affect the Business Cycle? — A Theoretical Consideration

Tokihito Sudo *

Abstract

This study describes the development of a new-Keynesian dynamic stochastic general equilibrium (DSGE) model in which market-based intermediaries or active investors have an interactive relationship with the ultimate borrowers and lenders.

The theoretical analysis yields two important propositions related to market-based intermediaries: first, the greater the active investor's asset size, the higher will be the expected net profits; second, the steeper the yield curve, the greater is the asset size. These propositions together suggest that steeper yield curves will yield higher net profits to active investors, which theoretically supports the views of Adrian, Moench and Shin [2010b], Cúrdia and Woodford [2010] and Woodford [2010].

Next step is to perform empirical analyses based on the developed model: to examine the propositions derived in this study, and to assess effects of the presence of the market-based intermediary sector on the business cycle. We will present the empirical study in the near future.

Contents

* Senior researcher at the Japan Securities Research Institute; email: sudot@jsri.or.jp

I Introduction

It appears that since 2000, market-based intermediaries have played a crucial role in business cycles, especially boom-bust cycles, because nonbank financial intermediaries have become increasingly important sources of credit, particularly due to the growing popularity of securitization, as pointed out by many studies such as Adrian and Shin [2010a, 2011a] and Woodford [2010]. In that case, is it possible that the market-based intermediaries substantially impact business cycles? How would they do so? The research conducted on this issue so far has mainly focused on the behaviour of these intermediaries[1).

Recent studies tend to focus more on developing new-Keynesian macroeconomic models or dynamic stochastic general equilibrium (DSGE) models, wherein financial institutions play a crucial role and which allow for frictions that can impede an efficient supply of credit. Woodford [2010], Adrian and Shin [2011a], and Gertler and Kiyotaki [2011] provide surveys of the recent work in this area.

These models could be roughly classified into three groups. The first group focuses on financial frictions that arise from the behaviour of borrowers, but does not consider the behaviour of financial institutions themselves. There are many studies that can be categorized in this group, for example, Bernanke, Gertler and Gilchrist [1999], Christiano, Motto and Rostagno [2003, 2008, 2010], Aliaga-Diaz and Olivero [2007], Goodfriend and McCallum [2007], Kobayashi [2008], Teranishi [2008], Andres and Arce [2009], and Gilchrist, Ortiz and Zakrajsek [2009].

The second group takes into account the influences of the behaviours of both financial intermediaries and borrowers on an economy; this group of studies includes Cúrdia and Woodford [2010], Gerali et al. [2010], Iacoviello and Neri [2010], and Verona, Martins and Drumond [2011]. However, the models in this group do not allow for the possibility that the intermediaries' behaviour will be affected by changes in the price of assets which they hold, although some of models regard the effects of changes in the value of assets pledged by borrowers as collateral-for example, houses and capital-on the behaviour of intermediaries.

The third group considers all the factors stated above, that is, the effects of not only the behaviour of borrowers and financial institutions but also of financial asset markets on the economy. The research carried out by Adrian, Moench and Shin [2010b] falls under this group. In their study, the researchers attempt to extend the standard new-Keynesian macroeconomic model by introducing the concepts of the macro risk premium and the risk appetite relevant to market-based intermediaries[2). Their idea is derived from their vigorous studies related to market-based intermediaries and definitely contributes to the development of the macroeconomic model. However, it is regrettable that the relationships among the macro risk premium, the risk appetite and other macro variables such as real output and interest rates are specified ad hoc and therefore lack a microeconomic foundation.

We have two objectives: first, to present a new-Keynesian DSGE model which will provide a microeconomic foundation to the interconnections among the ultimate borrowers, ultimate lenders and market-based financial intermediaries; second, to perform empirical analyses, using the U.S. data, and to assess the influences of the market-based intermediary sector on business cycles. This paper tries to attain the first objective.

II Model setup

In this section, we will consider a model economy composed of end-user borrowers, active investors, passive investors, firms and the central bank, and describe the behaviour of these agents. The framework of the economy follows that suggested by Adrian, Moench and Shin [2010b].

The starting point for our analysis is a hybrid new-Keynesian DSGE model with sticky prices and habits in consumption. Furthermore, we have further developed the standard model by making several modifications as follows. First, it is assumed that there are three different types of financial instruments-deposits, short-term bonds and long-term bonds. For trading in bonds, we have introduced a financial friction that makes these different types of bonds imperfect substitutes, as in Andrés, López-Salido and Nelson [2004a, b] (henceforth, ALSN), Marzo, Söderström and Zagaglia [2007], and Sudo [2010]. The friction reveals the endogenous term structure of interest rates in the sense that there exists bi-directional feedback between the yield curve and the economy.

Second, we will pay attention to the roles played by financial intermediary mechanisms and leveraged mechanisms in the economy, similar to Adrian, Moench and Shin [2010b]. Active investors play the role of intermediaries between end-user borrowers and passive investors. In addition, they issue short-term bonds to purchase more long-term bonds while complying with a minimum capital requirement. In this process, they make use of leveraged mechanisms such as shadow banks in order to maximize their net profits[3]. The leveraged mechanisms will be represented as a relationship between the active investor's assets and the endogenous term structure of interest rates.

Third, a collateral constraint has been incorporated into the model. While end-user borrowers as well as active investors raise funds by issuing bonds, the issues should be secured by using their assets as collateral, as suggested by Kiyotaki and Moore [1997], Pintus and Wen [2008], Gerali et al. [2010], and Iacoviello and Neri [2010].

Finally, the model considers the concept of probability of default. For this, we can presuppose two financial states of the economy: the first is the economy's state during stable times, while the second is its state during financial distress. Which state the economy is in depends on whether or not the end-user borrowers reach a situation of financial distress, which will have chain impacts on the behaviour of other agents, particularly the behaviour of active and passive investors.

In the following sections, we will present the objectives and constraints of different agents in the economy, paying special attention to specifying the behaviour of end-user borrowers as well as active and passive investors.

Ⅲ End-user borrowers

1. Utility function and constraints

End-user borrowers basically behave like households; thus, they derive incomes from labour and consume goods. In this analysis, we will assume a continuum of identical and infinitely living borrowers, indexed by $i_1 \in [0, 1]$, and a continuum of consumption goods, indexed by $j \in [0, 1]$, which are produced by the firm j. These borrowers obtain utility from a bundle $C_{1,t}$, given by

$$C_{1,t}=\left[\int_0^1 C_{1,t}(j)^{1-\frac{1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where $C_{1,t}(j)$ represents the quantity of goods j consumed by the borrower in period t, and ε is the elasticity of substitution across different varieties of goods. Moreover, during stable times, the borrowers invest in housing by issuing long-term bonds which are zero-coupon bonds, and thus obtain utility from their residential holdings as well.

As a result, during stable times, these borrowers have the period utility function of

$$U_{1,p}(C_{1,t}, C_{1,t-1}, N^s_{1,t}, S_{h,t}, e_{h,t}) = \frac{1}{1-\sigma_{1,p}}\left(\frac{C_{1,t}}{C^{h_1}_{1,t-1}}\right)^{1-\sigma_{1,p}} + \frac{S^{1-\chi}_{h,t}}{1-\chi}e_{h,t} - \frac{(N^s_{1,t})^{1+\varphi_{1,p}}}{1+\varphi_{1,p}}, \tag{2}$$

where the subscript 'p' stands for stable times. In the utility function, $S_{h,t}$ denotes housing stock holdings at the beginning of period t; $N^s_{1,t}$, hours worked by the borrower in period t; $\sigma_{1,p}>0$ inverse of the elasticity of inter-temporal substitution; $\chi \geq 0$, inverse of the interest elasticity of the demand for housing; $\varphi_{1,p} \geq 0$, inverse of the Frisch labour supply elasticity; and $h_1 \geq 0$, the habit persistence parameter indicating the extent of habit formation. $e_{h,t}$ represents the shocks to the borrower's demand for housing in period t and follows the process

$$e'_{h,t}=\rho_h e'_{h,t-1}+\varepsilon_{h,t}, \tag{3}$$

where $e'_{h,t}\equiv\ln(e_{h,t})$, $\rho_h\in(0,1)$ and $\varepsilon_{h,t}\sim i.i.d.(0,\sigma_h^2)$.

The period budget constraint takes the form of

$$\frac{1}{P_t}\{W_tN_{1,t}^s+B_{l,t}^s+Q_{h,t}(1-\iota_h)S_{h,t-1}\}=C_{1,t}+\frac{1}{P_t}(H_{l,t-1}B_{l,t-1}^s+Q_{h,t}S_{h,t}), \tag{4}$$

where P_t stands for the price of the consumption goods in period t; W_t, the nominal wage in period t; $B_{l,t}^s$, long-term bond outstandings at the beginning of period t; $\iota_h\in(0,1)$, the depreciation rate on housing stock; and $Q_{h,t}$, house prices in period t. $H_{l,t}$ represents the gross one-period nominal costs of issuing the long-term zero-coupon bond and is calculated by $H_{l,t}=\frac{Q_{l,t+1}}{Q_{l,t}}$, where $Q_{l,t}$ represents the price of the long-term bond in period t. The budget constraint (4) implies that, at the beginning of period t, borrowers sell their own house at price $Q_{h,t}$ and repurchase long-term bonds issued in previous periods at price $Q_{l,t}$; at the same time, they issue the bond and purchase a new house at the same prices, respectively[4].

At this point, we introduce a collateral constraint, related to the issuances of long bonds, as follows:

$$\frac{1}{P_t}R_tB_{l,t}^s\leq\frac{1}{P_t}k_hQ_{h,t}S_{h,t}, \tag{5}$$

where R_t is the gross nominal one-period interest rate and $k_h\in(0,1]$ measures the collateral value of houses owned by the borrower. Since, as shown below, R_t represents the interest rates which are applied to the deposits of passive investors and the reserve of active investors, $R_tB_{l,t}^s$ indicates the minimum expected returns that investors would require on long-term bond purchases. Consequently, the constraint (5) implies that the minimum required returns on long-term bonds investment should be secured by the collateral value of assets owned by the borrower. In other words, the constraint states that the amount of debt cannot exceed the collateral value of the borrower's assets discounted by the risk-free rate.

On the other hand, when borrowers are faced with financial distress with probability X_t in period t, they have to sell their houses at price $Q_{h,t}$ and redeem their previously issued long-term bonds $B_{l,t-1}^s$. This is expressed by the equation

$$Q_{h,t}(1-\iota_h)S_{h,t-1}=\gamma H_{l,t-1}B_{l,t-1}^s, \tag{6}$$

where $\gamma\in(0,1]$ denotes the rate at which borrowers can redeem long-term bonds by disposing of their own houses. Afterwards, during subsequent financial distress periods, the borrower will not purchase a house.

Consequently, when the economy is in a state of financial distress, the period utility function is

$$U_{1,d}(C_{1,t},C_{1,t-1},N_{1,t}^s)=\frac{1}{1-\sigma_{1,d}}\left(\frac{C_{1,t}}{C_{1,t-1}^{h_1}}\right)^{1-\sigma_{1,d}}-\frac{\left(N_{1,t}^s\right)^{1+\varphi_{1,d}}}{1+\varphi_{1,d}}, \tag{7}$$

where the subscript '*d*' represents times of financial distress. The budget constraint is described as

$$\frac{1}{P_k}\{W_k N^s_{1,k}+Q_{h,k}(1-\iota_h)S_{h,k-1}\}=C_{1,k}+\frac{1}{P_k}\gamma H_{l,k-1}B^s_{l,k-1},\ \text{if } k=t,$$

and

$$\frac{1}{P_k}W_k N^s_{1,k}=C_{1,k},\ \text{if } k\geq t+1.$$

Using equation (6), the budget constraint in financial distress is described by

$$\frac{1}{P_t}W_t N^s_{1,t}=C_{1,t}, \tag{8}$$

which indicates that end-user borrowers face liquidity constraints during times of financial distress.

2. Optimality conditions and analyses

According to the above setup, the end-user borrower solves the following problem:

$$\max_{\{C_{1,t},N^s_{1,t},S_{h,t},B^s_{l,t}\}} E_t\sum_{i=0}^{\infty}\beta_1^{t+i}\Big\{\big(1-X_{t+i}\big)U_{1,p}\big(C_{1,t+i},C_{1,t+i-1},N^s_{1,t+i},S_{h,t+i},e_{h,t+i}\big)$$

$$+X_{t+i}U_{1,d}\big(C_{1,t+i},C_{1,t+i-1},N^s_{1,t+i}\big)\Big\},$$

$$\text{s.t. }\left(\frac{1}{P_{t+i}}\right)\Big[W_{t+i}N^s_{1,t+i}+(1-X_{t+i})\big\{B^s_{l,t+i}+(1-\iota_h)Q_{h,t+i}S_{h,t+i-1}\big\}\Big]$$

$$-C_{1,t+i}-\left(\frac{1}{P_{t+i}}\right)(1-X_{t+i})(H_{l,t+i-1}B^s_{l,t+i-1}+Q_{h,t+i}S_{h,t+i})=0 \tag{9}$$

and

$$\left(\frac{1}{P_{t+i}}\right)(1-X_{t+i})(k_h Q_{h,t+i}S_{h,t+i}-R_{t+i}B^s_{l,t+i})\geq 0, \tag{10}$$

where the parameter $\beta_1\in(0,\ 1)$ is a discount factor and E_t denotes the mathematical expectations operator conditional on information available in period t. In what follows, for any variable Z_t, $\overline{Z}$ stands for a steady state of the sequence $\{Z_t\}$ and Z'_t represents a real variable in terms of consumer price, that is, $Z'_t\equiv Z_t/P_t$.

Assuming that $\sigma_{1,p}=\sigma_{1,d}=\sigma_1$ and , $\varphi_{1,p}=\varphi_{1,d}=\varphi_1$, the first-order conditions for the optimizing problem given above can be written, in real terms, as follows:

$$\Lambda_{1,t}=\frac{C_{1,t}^{-\sigma_1}}{C_{1,t-1}^{h_1(1-\sigma_1)}}-\beta_1 h_1 E_t\left[\frac{C_{1,t+1}^{1-\sigma_1}}{C_{1,t}^{h_1(1-\sigma_1)+1}}\right], \tag{11}$$

$$\left(N^s_{1,t}\right)^{\varphi_1}=\Lambda_{1,t}W'_t, \tag{12}$$

$$S_{h,t}^{-\chi}e_{h,t}=(\Lambda_{1,t}-k_h\Lambda_{2,t})Q'_{h,t}-\beta_1(1-\iota_h)E_t\left[\Lambda_{1,t+1}Q'_{h,t+1}\left(\frac{1-X_{t+1}}{1-X_t}\right)\right], \tag{13}$$

$$\Lambda_{1,t}-\Lambda_{2,t}R_t=\beta_1 E_t\left[\Lambda_{1,t+1}H_{l,t}\left(\frac{1}{\Pi_{t+1}}\right)\left(\frac{1-X_{t+1}}{1-X_t}\right)\right], \tag{14}$$

$$C_{1,t}=W'_t N^s_{1,t}+(1-X_t)\left\{B^{s\prime}_{l,t}+(1-\iota_h)Q'_{h,t}S_{h,t-1}-H_{l,t-1}\frac{B^{s\prime}_{l,t-1}}{\Pi_t}-Q'_{h,t}S_{h,t}\right\}, \tag{15}$$

$$B^{s\prime}_{l,t}=k_h\left(\frac{Q'_{h,t}}{R_t}\right)S_{h,t}, \tag{16}$$

where, $\Pi_t \equiv P_t/P_{t-1}$ and $\Lambda_{1,t}$ and $\Lambda_{2,t}$ represent the Lagrange multipliers for the budget constraint and the collateral constraint, respectively. As shown by Pintus and Wen [2008], it can be proved that, as indicated by equation (16), the collateral constraint is binding around a steady state, under a condition imposed on the discount factors of the end-user borrower and the passive investor.

Considering a steady state of the economy, equations (12) and (14) imply the following, respectively,

$$\bar{\Lambda}_1=\left(\bar{N}^s_1\right)^{\varphi_1}\left(\bar{W}'\right)^{-1}>0, \tag{17}$$

$$\bar{\Lambda}_2=\frac{1-\beta_1\bar{H}_l}{\bar{R}}\bar{\Lambda}_1. \tag{18}$$

As described in section V, a passive investor's discount factor, β_2, is assumed to be larger than β_1 and equal to $\bar{H}_l^{-1}$ in a steady state. Then,

$$1-\beta_1\bar{H}_l=1-\beta_1\beta_2^{-1}=\beta_2^{-1}(\beta_2-\beta_1)>0;$$

hence, equations (17) and (18) indicate that

$$\bar{\Lambda}_2>0,$$

which implies that the collateral constraint is binding around the steady state[5].

We can derive the following proposition from the above first-order conditions:

Proposition 1: Weak or no collateral constraint would push up the housing stock (i.e. residential investments) in a steady state.

This proposition can be proved as follows. In a steady state, equation (13) implies that

$$\bar{S}_h^{-\chi}=\left[\{1-(1-\iota_h)\beta_1\}\bar{\Lambda}_1-k_h\bar{\Lambda}_2\right]\bar{Q}'_h. \tag{19}$$

Then,

$$\frac{\partial\bar{S}_h}{\partial k_h}=\frac{\bar{S}_h^{1+\chi}}{\chi}\bar{Q}'_h\bar{\Lambda}_2>0$$

because $\bar{\Lambda}_2>0$.

Furthermore, substituting equation (18) into equation (19), we obtain

$$\overline{S}_h^{-\chi}=\left\{1-(1-\iota_h)\beta_1-k_h\left(\frac{1-\beta_1\overline{H}_l}{\overline{R}}\right)\right\}\overline{Q}_h\overline{\Lambda}_1. \tag{20}$$

On the other hand, when there is no collateral constraint, the following equation can be derived from the first-order conditions in the steady state[6]:

$$\left(\overline{S}_h^*\right)^{-\chi}=\left\{1-(1-\iota_h)\frac{1}{\overline{H}_l}\right\}\overline{Q}_h\overline{\Lambda}_1, \tag{21}$$

where $\overline{S}_h^*$ stands for the steady state housing stock without the collateral constraint. According to equations (20) and (21),

$$\overline{S}_h^{-\chi}-\left(\overline{S}_h^*\right)^{-\chi}=\frac{1-\beta_1\overline{H}_l}{\overline{R}}\left\{(1-\iota_h)\frac{\overline{R}}{\overline{H}_l}-k_h\right\},$$

which implies that if k_h is small enough to meet $k_h<(1-\iota_h)\frac{\overline{R}}{\overline{H}_l}$, then $\overline{S}_h^*>\overline{S}_h$[7].

The above argument indicates that in the case of weak or zero collateral constraints, borrowers may issue more long-term bonds and increase their housing investment, which would lead to financial as well as economic instability.

Ⅳ Active investors

1. Net profits and constraints

Active investors consist of banks, security broker-dealers and shadow banks such as asset-backed security (ABS) issuers; hence, they play the dual roles of financial intermediaries and leveraged investors. In these two roles, they raise funds through deposits and issues of short-term bonds (ABS) to passive investors and allocate these funds in the form of reserves for the central bank and investments in long-term bonds issued by end-user borrowers. Here, we can assume that the short-term bond is a zero-coupon bond.

According to the active investor's behaviour, during stable times, the active investor's (expected) net profits in period t are defined as

$$F_p\left(B_{r,t},B_{1,l,t}^d,B_{s,t}^s\right)=(R_t-1)B_{r,t}+(H_{l,t}-1)B_{1,l,t}^d-(R_t-1)M_t-(H_{s,t}-1)B_{s,t}^s, \tag{22}$$

where $B_{r,t}$ represents reserve outstandings at the beginning of period t; $B_{1,l,t}^d$ represents holdings of long-term bonds at the beginning of period t; and $B_{s,t}^s$ stands for short-term bond outstandings at the beginning of period t. M_t, the value of which is given, represents the deposits accepted from passive investors at the beginning of period t. $H_{s,t}$ stands for the gross one-period nominal cost of issuing short-term bonds and is calculated by $H_{s,t}=\frac{Q_{s,t+1}}{Q_{s,t}}$, where $Q_{s,t}$ is the price of the short bond in period t[8]. In equation (22), it should be noted that the central bank grants the same interest rates to its reserves as to its deposits.

Furthermore, it is supposed that active investors are risk-neutral but face two constraints: a budget constraint and a collateral constraint. The budget constraint is expressed as

$$R_tB_{r,t}(1+AC_{l,t})+H_{l,t}B^d_{1,l,t}-R_tM_t-H_{s,t}B^s_{s,t}=E_m, \tag{23}$$

where E_m is the minimum capital (capital adequacy) requirement[9),10)]. $AC_{l,t}$, which denotes the cost function for investing in long-term bonds, is specified as follows:

$$AC_{l,t}\equiv\frac{v_{1,l}}{2}\left(\frac{B_{r,t}}{B^d_{1,l,t}}\kappa_{1,l}-1\right)^2, \tag{24}$$

where $v_{1,l}>0$ and $\kappa_{1,l}\equiv\frac{\overline{B}^d_{1,l}}{\overline{B}_r}>0$ are parameters and $\kappa_{1,l}$ ensures that the cost $AC_{l,t}$ remains steady[11)]. The implications of the cost function are that active investors perceive long-term bonds as riskier assets, entailing a loss of liquidity in relation to their reserves. When active investors invest in long-term bonds, they demand additional reserves (liquidity) to compensate themselves for the loss of liquidity. In other words, the agents have self-imposed 'liquidity requirements' of $R_t(B_{r,t}\times AC_{l,t})$ from their long-term bond investment, where, because deposits are redeemed with interests, it is assumed that the agents reserve the liquidity requirement multiplied by the same interest rate, R_t, as deposits[12)].

The collateral constraint has the same implications as that on long-term bond issues-that is, short-term bond issues should be secured by the active investor's assets. Therefore, the collateral constraint is described as

$$R_tB^s_{s,t}\leq k_{bs}H_{l,t}B^d_{1,l,t}, \tag{25}$$

where $k_{bs}\in(0,1]$ measures the collateral value of the long-term bond holdings.

On the other hand, when end-user borrowers face financial distress with probability X_t, this exerts a bad influence on the active investor's net profits and budget constraints. Based on equation (6), if borrowers face financial distress in period t, the active investors would receive $\gamma H_{l,t}B^d_{1,l,t}$ instead of $H_{l,t}B^d_{1,l,t}$. In that case, active investors would redeem all the previously issued short-term bonds by an amount equal to $k_{bs}\gamma H_{l,t}B^d_{1,l,t}$, according to the collateral agreement (25).

Consequently, in the state of financial distress, the active investor's (expected) net profits in period t are

$$F_d(B_{r,t},B^d_{1,l,t},B^s_{s,t})=(R_t-1)B_{r,t}+(\gamma H_{l,t}-1)B^d_{1,l,t}-(R_t-1)M_t-(k_{bs}\gamma H_{l,t}B^d_{1,l,t}-B^s_{s,t}). \tag{26}$$

In addition, the budget constraint in period t is given by

$$R_tB_{r,t}(1+AC_{l,t})+\gamma H_{l,t}B^d_{1,l,t}-R_tM_t-k_{bs}\gamma H_{l,t}B^d_{1,l,t}+L_t=E_m, \tag{27}$$

where L_t denotes capital injections through the central bank to maintain the minimum capital requirement.

Thereafter, during periods of financial distress, active investors only accept deposits and put them in reserve accounts in the central bank. From these, they would gain net profits

$(R_{t+i}-1)(B_{r,t+i}-M_{t+i})=(R_{t+i}-1)E_m$ $(i\geq 1)$ and repay the injected capital L_t by using these net profits. Therefore, the active investor's net profits continue to be zero during such periods.

2. Optimality conditions and analyses

Based on the above setup, the active investor solves the following problem:

$$\max_{\{B_{r,t},B^d_{1,l,t},B^s_{s,t}\}} (1-X_t)F_p(B_{r,t},B^d_{1,l,t},B^s_{s,t})+X_tF_d(B_{r,t},B^d_{1,l,t},B^s_{s,t})$$

s.t.

$$R_tB_{r,t}(1+AC_{l,t})-R_tM_t+(1-X_t)(H_{l,t}B^d_{1,l,t}-H_{s,t}B^s_{s,t})+X_t\{\gamma(1-k_{bs})H_{l,t}B^d_{1,l,t}+L_t\}=E_m \tag{28}$$

and

$$(1-X_t)(k_{bs}H_{l,t}B^d_{1,l,t}-R_tB^s_{s,t})\geq 0. \tag{29}$$

The first-order conditions for the optimizing problem are described, in real terms, as follows:

$$-\Lambda_{3,t}R_t\left\{1+\frac{v_{1,l}}{2}\left(\frac{B'_{r,t}}{B^{d\prime}_{1,l,t}}\kappa_{1,l}-1\right)^2+v_{1,l}\kappa_{1,l}\left(\frac{B'_{r,t}}{B^{d\prime}_{1,l,t}}\kappa_{1,l}-1\right)\left(\frac{B'_{r,t}}{B^{d\prime}_{1,l,t}}\right)\right\}=R_t-1, \tag{30}$$

$$(1-X_t)\Lambda_{4,t}R_t=1-(1-X_t)(1+\Lambda_{3,t})H_{s,t}, \tag{31}$$

$$-v_{1,l}\kappa_{1,l}\Lambda_{3,t}R_t\left(\frac{B'_{r,t}}{B^{d\prime}_{1,l,t}}\kappa_{1,l}-1\right)\left(\frac{B'_{r,t}}{B^{d\prime}_{1,l,t}}\right)^2=1-\{(1-X_t)(1+\Lambda_{3,t}+k_{bs}\Lambda_{4,t})+\gamma(1-k_{bs})X_t(1+\Lambda_{3,t})\}H_{l,t}, \tag{32}$$

$$\begin{aligned}R_tB'_{r,t}\left\{1+\frac{v_{1,l}}{2}\left(\frac{B'_{r,t}}{B^{d\prime}_{1,l,t}}\kappa_{1,l}-1\right)^2\right\}=E'_m+R_tM'_t+(1-X_t)(H_{s,t}B^{s\prime}_{s,t}-H_{l,t}B^{d\prime}_{1,l,t})\\-\gamma(1-k_{bs})X_tB^{d\prime}_{1,l,t}-X_tL'_t,\end{aligned} \tag{33}$$

$$B^{s\prime}_{s,t}=k_{bs}\left(\frac{H_{l,t}}{R_t}\right)B^{d\prime}_{1,l,t}, \tag{34}$$

where $\Lambda_{3,t}$ and $\Lambda_{4,t}$ represent the Lagrange multipliers for the budget constraint and the collateral constraint, respectively. It can be proved that, as shown in equation (34), the collateral constraint is binding around a steady state under the condition of $\overline{R}>(1-\overline{X})\overline{H}_s$, based on equations (30) and (31)[13].

At this point, we should pay attention to equation (30). Denoting $\Gamma(B_{r,t}, B^d_{1,l,t}, B^s_{s,t}, \Lambda_{3,t}, \Lambda_{4,t})$ as the Lagrangean of the optimizing problem, which implicitly represents the active investor's expected net profits, we can show that $\frac{\partial\Gamma}{\partial E_m}=-\Lambda_{3,t}$. This indicates that $-\Lambda_{3,t}$ could be interpreted as the shadow value of the active investor's capital (i.e. equity). Meanwhile, equation (30) yields

$$-\Lambda_{3,t}=\left(\frac{R_t-1}{R_t}\right)\left(\frac{1}{V_t}\right), \tag{35}$$

where

$$V_t \equiv 1+\frac{v_{1,l}}{2}\left(\frac{B'_{r,t}}{B^{d'}_{1,l,t}}\kappa_{1,l}-1\right)^2+v_{1,l}\kappa_{1,l}\left(\frac{B'_{r,t}}{B^{d'}_{1,l,t}}\kappa_{1,l}-1\right)\left(\frac{B'_{r,t}}{B^{d'}_{1,l,t}}\right) \quad (36)$$

Setting $Z_t=\left(\frac{B'_{r,t}}{B^{d'}_{1,l,t}}\right)\kappa_{1,l}$,

$$V_t=\frac{v_{1,l}}{2}(Z_t-1)^2+v_{1,l}Z_t(Z_t-1)+1=\frac{3v_{1,l}}{2}\left(Z_t-\frac{2}{3}\right)^2+\frac{1}{6}(6-v_{1,l}). \quad (37)$$

Consequently, for V_t to be positive for all t, the condition $0<v_{1,l}<6$ must hold, which is quite probable. Furthermore,

$$\frac{dV_t}{dZ_t}=3v_{1,l}\left(Z_t-\frac{2}{3}\right) \quad (38)$$

and

$$\frac{d^2V_t}{dZ_t^2}=3v_{1,l}>0. \quad (39)$$

In equation (38), if $Z_t<\frac{2}{3}$, that is, if $B^{d'}_{1,l,t}>\frac{3}{2}\kappa_{1,l}B'_{r,t}$, then $\frac{dV_t}{dZ_t}<0$. This result, together with equations (35), (37) and (39), suggests the following proposition.

Proposition 2: Under the condition of $0<v_{1,l}<6$, the marginal increases in capital (i.e. equity) would raise the expected net profits. Although rises in the active investor's risky assets, $B^d_{1,l,t}$, might lead to increases in capital, the larger risky assets would diminish the marginal increases in the net profits.

This proposition implies that the growth of the active investor's assets would push up the (expected) net profits under the abovementioned condition on $v_{1,l}$.

Furthermore, from the first-order conditions, we can derive a very important feature of the model. This feature is stated as proposition 3.

Proposition 3: When $X_t=\overline{X}$ for all t and $\overline{X}>1-\frac{\overline{R}}{H_s}$, the active investor's holdings of risky assets are affected by the slope of the yield curve, and hence, the steeper the yield curve, the larger is its asset size[14].

This proposition can be proved as follows. In what follows, for any variables Z_t, we can define

$z_t\equiv\ln(Z_t)$.

Furthermore, with all variables z_t defined as above, we can define

$\hat{z}_t\equiv z_t-\bar{z}$,

where $\bar{z}\equiv\ln(\overline{Z})$.

Log-linear approximations of equations (30) to (32) with $X_t=\overline{X}$ for all t yield the following:

$$\hat{\lambda}_{3,t}=\frac{1}{\bar{R}-1}\hat{r}_t+v_{1,l}\left(\hat{b}^d_{1,l,t}-\hat{b}_{r,t}\right), \tag{40}$$

$$\hat{\lambda}_{4,t}=-\hat{r}_t+\frac{(1-\bar{X})\bar{H}_s}{\bar{R}-(1-\bar{X})\bar{H}_s}\left\{(\bar{R}-1)\hat{\lambda}_{3,t}-\hat{h}_{s,t}\right\}, \tag{41}$$

$$\begin{aligned}(\bar{R}-1)v_{1,l}\left(\frac{\bar{B}'_r}{\bar{B}^{d\prime}_{1,l}}\right)\left(\hat{b}^d_{1,l}-\hat{b}_{r,t}\right)&=-\hat{h}_{l,t}+\{(1-\bar{X})+\gamma(1-k_{bs})\bar{X}\}(\bar{R}-1)\left(\frac{\bar{H}_l}{\bar{R}}\right)\hat{\lambda}_{3,t}\\&\quad-k_{bs}\left\{\frac{\bar{R}-(1-\bar{X})\bar{H}_s}{\bar{R}}\right\}\left(\frac{\bar{H}_l}{\bar{R}}\right)\hat{\lambda}_{4,t}.\end{aligned} \tag{42}$$

Substituting equations (40) and (41) into equation (42) and rearranging the resultant equation, we obtain

$$(\bar{R}-1)v_{1,l}\left(1-k_{bs}\frac{\bar{H}_l}{\bar{R}}-\frac{\bar{B}'_r}{\bar{B}^{d\prime}_{1,l}}\right)\left(\hat{b}^d_{1,l}-\hat{b}_{r,t}\right)=\left(\hat{h}_{l,t}-\hat{r}_t\right)-k_{bs}\frac{(1-\bar{X})\bar{H}_s}{\bar{R}}\left(\frac{\bar{H}_l}{\bar{R}}\right)\left(\hat{h}_{s,t}-\hat{r}_t\right). \tag{43}$$

Equation (34) implies that

$$k_{bs}=\frac{\bar{R}\,\bar{B}^{s\prime}_s}{\bar{H}_l\bar{B}'^{d}_{1,l}}. \tag{44}$$

Hence, substituting equation (44) into equation (43), we obtain

$$(\bar{R}-1)v_{1,l}\left(1-\frac{\bar{B}^{s\prime}_s}{\bar{B}^{d\prime}_{1,l}}-\frac{\bar{B}'_r}{\bar{B}^{d\prime}_{1,l}}\right)\left(\hat{b}^d_{1,l}-\hat{b}_{r,t}\right)=\left(\hat{h}_{l,t}-\hat{r}_t\right)-k_{bs}\frac{(1-\bar{X})\bar{H}_s}{\bar{R}}\left(\frac{\bar{H}_l}{\bar{R}}\right)\left(\hat{h}_{s,t}-\hat{r}_t\right). \tag{45}$$

When $\bar{H}_l$, $\bar{H}_s$ and $\bar{R}$ are approximately equivalent, the budget constraint (33) implies that

$$\begin{aligned}\bar{B}^{d\prime}_{1,l}-\bar{B}^{s\prime}_s-\bar{B}'_r&\approx\bar{H}_l\bar{B}^{d\prime}_{1,l}-\bar{H}_s\bar{B}^{s\prime}_s-\bar{R}\,\bar{B}'_r\\&=\frac{1}{1-\bar{X}}\left\{\bar{E}'_m+\bar{R}\left(\bar{M}'-2\bar{B}'_r\right)\right\}-\frac{1}{1-\bar{X}}\left\{\gamma(1-k_{bs})\bar{B}^{d\prime}_{1,l}+\bar{L}'-\bar{R}\,\bar{B}'_r\right\}.\end{aligned} \tag{46}$$

Because $\bar{M}'>2\bar{B}'_r$ would undoubtedly hold, equation (46) results in $\bar{B}^{d\prime}_{1,l}-\bar{B}^{s\prime}_s-\bar{B}'_r>0$ when $\bar{X}$ is substantially small. Therefore, equations (45) and (46) bear out proposition 3. The proposition theoretically establishes a relationship between 'the macro risk premium' and 'the risk appetite' advocated by Adrian, Moench and Shin [2010b], where the macro risk premium and the risk appetite correspond to the slope of the yield curve and the active investor's asset size, respectively, in the proposition[15].

V Passive investors

1. Utility function and budget constraint

Passive investors consist of households other than the end-user borrowers and institutional investors such as mutual funds, pension funds and insurance companies; hence, passive investors play the roles of consumers as well as funding sources in the economy.

In this study, we presume a continuum of identical and infinitely living passive investors indexed by $j_2 \in [0, 1]$. These investors obtain utility from a bundle $C_{2,t}$ given by

$$C_{2,t}=\left[\int_0^1 C_{2,t}(j)^{1-\frac{1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{47}$$

where $C_{2,t}(j)$ denotes the quantity of goods j consumed by the investor in period t.

These passive investors have the period utility function of

$$U_{2,k}(C_{2,t},C_{2,t-1},N^s_{2,t})=\frac{1}{1-\sigma_{2,k}}\left(\frac{C_{2,t}}{C^{h_2}_{2,t-1}}\right)^{1-\sigma_{2,k}}-\frac{\left(N^s_{2,t}\right)^{1+\varphi_{2,k}}}{1+\varphi_{2,k}}, \tag{48}$$

where $k=p$ (stable times) or d(financial distress). $N^s_{2,t}$ represents the number of hours worked by the investor in period t; $\sigma_{2,k}>0$, the inverse of the elasticity of inter-temporal substitution; $\varphi_{2,k}\geq 0$, the inverse of the Frisch labour supply elasticity; and $h_2\geq 0$, the habit persistence parameter indicating the extent of habit formation.

In stable times, passive investors allocate their incomes for the purchase of short- and long-term bonds as well as consumption goods and deposits. Consequently, the period budget constraint in stable times takes the form of

$$\frac{1}{P_t}(W_tN^s_{2,t}+H_{l,t-1}B^d_{2,l,t-1}+H_{s,t-1}B^d_{s,t-1}+R_{t-1}M_{t-1}+T_t)=C_{2,t}+\frac{1}{P_t}\{B^d_{2,l,t}+B^d_{s,t}+M_t(1+AC_{m,t})\}, \tag{49}$$

where $B^d_{2,l,t}$ denotes the holdings of long-term bonds at the beginning of period t; $B^d_{s,t}$, the holdings of short-term bonds at the beginning of period t; and M_t, the deposit amounts, which imply demand for money, at the beginning of period t. T_t represents lump sum transfers, which include dividends from firms and active investors of which the passive investors are the only owners, and is derived by

$$t_t=\rho_t t_{t-1}+\varepsilon_{t,t}, \tag{50}$$

where $t_t\equiv\ln(T_t)$, $\rho_t \in (0, 1)$ and $\varepsilon_{t,t}\sim i.i.d.(0,\sigma_t^2)$. $AC_{m,t}$ is the cost function for investing in short- and long-term bonds and is specified as follows:

$$AC_{m,t}\equiv\frac{\upsilon_{2,s}}{2}\left(\frac{M_t}{B^d_{s,t}}\kappa_{2,s}-1\right)^2+\frac{\upsilon_{2,l}}{2}\left(\frac{M_t}{B^d_{2,l,t}}\kappa_{2,l}-1\right)^2, \tag{51}$$

where $\upsilon_{2,s}>0$ and $\upsilon_{2,l}>0$ are parameters. $\kappa_{2,s}>0$ and $\kappa_{2,l}>0$ are parameters ensuring that the cost $AC_{m,t}$ remains in the steady state; they are defined as $\kappa_{2,s}\equiv\overline{B}^d_s / \overline{M}$ and $\kappa_{2,l}\equiv\overline{B}^d_{2,l,t} / \overline{M}$.

The cost function has the same implication as that of the cost function $AC_{l,t}$ on the active investor's budget constraint (23). Hence, passive investors perceive both short- and long-term bonds as riskier assets, entailing a loss of liquidity in relation to their deposits. When passive investors invest in short- and long-term bonds, they demand additional money (i.e. deposits as risk-free assets) to compensate themselves for this loss of liquidity. In effect, the agents have self-imposed the 'reserve requirements' (as described in ALSN [2004a]) of $M_t \times AC_{m,t}$ on their short- and long-term bond investments[16].

On the other hand, when the end-user borrowers face financial distress in period $t-1$, they redeem the long-term bonds of $\gamma H_{l,t-1}B^s_{l,t-1}$ instead of repurchasing those of $H_{l,t-1}B^s_{l,t-1}$, and this causes active investors to repay the short-term bonds of $k_{bs}\gamma H_{l,t-1}B^d_{1,l,t-1}$ instead of $H_{s,t-1}B^d_{s,t-1}$, as explained above. As a result, in the state of financial distress, the passive investor budget constraint in period t is

$$\frac{1}{P_t}\left(W_t N^s_{2,t}+\gamma H_{l,t-1}B^d_{2,l,t-1}+k_{bs}\gamma H_{l,t-1}B^d_{1,l,t-1}+R_{t-1}M_{t-1}+T_t\right)=C_{2,t}+\frac{1}{P_t}M_t. \tag{52}$$

Afterwards, since passive investors will not invest in short- and long-term bonds, the budget constraint in period $t+i\,(i\geq 1)$ becomes

$$\frac{1}{P_{t+i}}\left(W_{t+i}N^s_{2,t+i}+R_{t+i-1}M_{t+i-1}+T_{t+i}\right)=C_{2,t+i}+\frac{1}{P_{t+i}}M_{t+i}. \tag{53}$$

2. Optimality conditions and analyses

According to the above setup, the problem solved by the passive investor can be expressed as follows:

$$\max_{\{C_{2,t},N^s_{2,t},B^d_{s,t},B^d_{2,l,t},M_t\}} E_t\sum_{i=0}^{\infty}\beta_2^{t+i}\Big\{\left(1-X_{t+i}\right)U_{2,p}\left(C_{2,t+i},C_{2,t+i-1},N^s_{2,t+i}\right)$$
$$+X_{t+i}U_{2,d}\left(C_{2,t+i},C_{2,t+i-1},N^s_{2,t+i}\right)\Big\}$$

$$\text{s.t.}\ \left(\frac{1}{P_{t+i}}\right)\Big\{W_{t+i}N^s_{2,t+i}+T_{t+i}+(1-X_{t+i})\left(H_{s,t+i-1}B^d_{s,t+i-1}+H_{l,t+i-1}B^d_{2,l,t+i-1}\right)$$
$$+\xi_{t+i}X_{t+i}\gamma H_{l,t+i-1}\left(k_{bs}B^d_{1,l,t+i-1}+B^d_{2,l,t+i-1}\right)+R_{t+i-1}M_{t+i-1}\Big\} \tag{54}$$
$$-C_{2,t+i}-\left(\frac{1}{P_{t+i}}\right)\Big\{M_{t+i}+(1-X_{t+i})\left(B^d_{s,t+i}+B^d_{2,l,t+i}+M_{t+i}AC_{m,t+i}\right)\Big\}=0,$$

where the parameter $\beta_2 \in (0, 1)$ is a discount factor and we suppose $\beta_2>\beta_1$ because the passive investor is a lender whereas the end-user borrower is a borrower, that is, the lender is likely to be more patient than the borrower. ξ_{t+i} denotes a dummy variable which takes the value of one if $i=0$ and zero otherwise.

Assuming that $\sigma_{2,p}=\sigma_{2,d}=\sigma_2$ and $\varphi_{2,p}=\varphi_{2,d}=\varphi_2$, the first-order conditions for the optimizing problem given above can be written, in real terms, as follows:

$$\Lambda_{5,t}=\frac{C_{2,t}^{-\sigma_2}}{C_{2,t-1}^{h_2(1-\sigma_2)}}-\beta_2 h_2 E_t\left[\frac{C_{2,t-1}^{1-\sigma_2}}{C_{2,t}^{h_2(1-\sigma_2)+1}}\right], \tag{55}$$

$$\left(N^s_{2,t}\right)^{\varphi_2}=\Lambda_{5,t}W'_t, \tag{56}$$

$$\Lambda_{5,t}-\beta_2 E_t\left[\Lambda_{5,t+1}H_{s,t}\left(\frac{1}{\Pi_{t+1}}\right)\left(\frac{1-X_{t+1}}{1-X_t}\right)\right]=\Lambda_{5,t}\left\{v_{2,s}\kappa_{2,s}\left(\frac{M'_t}{B^{d\prime}_{s,t}}\kappa_{2,s}-1\right)\left(\frac{M'_t}{B^{d\prime}_{s,t}}\right)^2\right\}, \quad (57)$$

$$\Lambda_{5,t}-\beta_2 E_t\left[\Lambda_{5,t+1}H_{l,t}\left(\frac{1}{\Pi_{t+1}}\right)\left(\frac{1-X_{t+1}}{1-X_t}\right)\right]=\Lambda_{5,t}\left\{v_{2,l}\kappa_{2,l}\left(\frac{M'_t}{B^{d\prime}_{2,l,t}}\kappa_{2,l}-1\right)\left(\frac{M'_t}{B^{d\prime}_{2,l,t}}\right)^2\right\}, \quad (58)$$

$$\Lambda_{5,t}-\beta_2 R_t E_t\left[\Lambda_{5,t+1}\left(\frac{1}{\Pi_{t+1}}\right)\right]$$
$$=-\Lambda_{5,t}(1-X_t)\left\{\frac{v_{2,s}}{2}\left(\frac{M'_t}{B^{d\prime}_{s,t}}\kappa_{2,s}-1\right)^2+\frac{v_{2,l}}{2}\left(\frac{M'_t}{B^{d\prime}_{2,l,t}}\kappa_{2,l}-1\right)^2\right.$$
$$\left.+v_{2,s}\kappa_{2,s}\left(\frac{M'_t}{B^{d\prime}_{s,t}}\kappa_{2,s}-1\right)\left(\frac{M'_t}{B^{d\prime}_{s,t}}\right)+v_{2,l}\kappa_{2,l}\left(\frac{M'_t}{B^{d\prime}_{2,l,t}}\kappa_{2,l}-1\right)\left(\frac{M'_t}{B^{d\prime}_{2,l,t}}\right)\right\}, \quad (59)$$

$$C_{2,t}=\left[W'_t N^s_{2,t}+T'_t+(1-X_t)\left(H_{s,t-1}\frac{B^{d\prime}_{s,t-1}}{\Pi_t}+H_{l,t-1}\frac{B^{d\prime}_{2,l,t-1}}{\Pi_t}\right)\right.$$
$$+\gamma X_t\xi_t H_{l,t}\left(k_{bs}\frac{B^{d\prime}_{1,l,t-1}}{\Pi_t}+\frac{B^{d\prime}_{2,l,t-1}}{\Pi_t}\right)+R_{t-1}\frac{M'_{t-1}}{\Pi_t}-M'_t \quad (60)$$
$$\left.-(1-X_t)\left\{B^{d\prime}_{s,t}+B^{d\prime}_{2,l,t}+M'_t\left\{\frac{v_{2,s}}{2}\left(\frac{M'_t}{B^{d\prime}_{s,t}}\kappa_{2,s}-1\right)^2+\frac{v_{2,l}}{2}\left(\frac{M'_t}{B^{d\prime}_{2,l,t}}\kappa_{2,l}-1\right)^2\right\}\right\}\right],$$

where $\Lambda_{5,t}$ represents the Lagrange multiplier for the budget constraint, and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the aggregate inflation rate in period t.

For the first-order conditions, it is important to pay attention to two points. First, according to equation (59), the existence of the reserve requirement entails that money demand decisions are taken based on the relative supply of riskier bonds. In particular, an increase in the relative amount of riskier assets correspondingly raises the demand for money as liquid or risk-free assets.

Second, equations (57) to (59) imply the presence of an endogenous term structure relationship between the one-period nominal interest rate and the one-period nominal holding return on short- or long-term bonds. This feature is very important and is stated as the following proposition.

Proposition 4: The term structure of interest rates, in terms of the term premium between the one-period nominal interest rate and the one-period nominal holding return on short- or long-term bonds, is endogenously shifted by the modified ratio of money to each bond holding.

This proposition can be proved as follows. The log-linear approximation of equations (57) to (59) gives us

$$E_t[\hat{\lambda}_{5,t+1}]-\hat{\lambda}_{5,t}+\hat{h}_{s,t}-E_t[\hat{\pi}_{t+1}]=-v_{2,s}\left(\frac{\overline{M'}}{\overline{B^{d\prime}_s}}\right)(\hat{m}_t-\hat{b}^d_{s,t})+\frac{\overline{X}}{1-\overline{X}}E_t[\hat{x}_{t+1}-\hat{x}_t], \quad (61)$$

$$E_t[\hat{\lambda}_{5,t+1}]-\hat{\lambda}_{5,t}+\hat{h}_{l,t}-E_t[\hat{\pi}_{t+1}]=-v_{2,l}\left(\frac{\overline{M}'}{\overline{B}^{d'}_{2,l}}\right)(\widehat{m}_t-\hat{b}^d_{2,l,t})+\frac{\overline{X}}{1-\overline{X}}E_t[\hat{x}_{t+1}-\hat{x}_t], \tag{62}$$

$$E_t[\hat{\lambda}_{5,t+1}]-\hat{\lambda}_{5,t}+\hat{r}_t-E_t[\hat{\pi}_{t+1}]=(1-\overline{X})\{v_{2,s}(\widehat{m}_t-\hat{b}^d_{s,t})+v_{2,l}(\widehat{m}_t-\hat{b}^d_{2,l,t})\}. \tag{63}$$

By combining equations (63) with equation (61) or (62), we obtain equations (64) and (65), respectively.

$$\hat{h}_{s,t}=\hat{r}_t-v_{2,s}\left(\frac{\overline{M}'}{\overline{B}^{d'}_{s}}\right)(\widehat{m}_t-\hat{b}^d_{s,t})$$
$$-(1-\overline{X})\{v_{2,s}(\widehat{m}_t-\hat{b}^d_{s,t})+v_{2,l}(\widehat{m}_t-\hat{b}^d_{2,l,t})\}+\frac{\overline{X}}{1-\overline{X}}E_t[\hat{x}_{t+1}-\hat{x}_t], \tag{64}$$

$$\hat{h}_{l,t}=\hat{r}_t-v_{2,l}\left(\frac{\overline{M}'}{\overline{B}^{d'}_{2,l}}\right)(\widehat{m}_t-\hat{b}^d_{2,l,t})$$
$$-(1-\overline{X})\{v_{2,s}(\widehat{m}_t-\hat{b}^d_{s,t})+v_{2,l}(\widehat{m}_t-\hat{b}^d_{2,l,t})\}+\frac{\overline{X}}{1-\overline{X}}E_t[\hat{x}_{t+1}-\hat{x}_t]. \tag{65}$$

It should be noted that one-period holding returns on the short- or long-term bond are affected by the demand not only for the corresponding bond but also for the other bond. In these equations, $\hat{r}_t$ is endogenously determined by the central bank, as explained in section Ⅳ. Therefore, these equations lead to proposition 4; hence, the entire shape of the yield curve in terms of the one-period holding returns is endogenously determined in the economy.

Proposition 4 has two implications. First, equations (64) and (65) capture an essential feature of Tobin's [1969] framework which maintains that spreads between interest rates should reflect the relative quantities of assets. Second, as shown in sub-section 2, the endogenous term structure would affect the active investor's holdings of risky assets, that is, the asset size. Hence, the endogenous term structure of interest rates can provide a bi-directional feedback between the active investor's behaviour and the real economy (economic activities).

Ⅵ Firms, the central bank, and complete model

1. Firms

In this sub-section, we will illustrate the derivation of the hybrid new-Keynesian Phillips curve (NKPC)[17].

As stated above, we assume a continuum of firms indexed by $j \in [0, 1]$. Each firm produces a differentiated consumption good $C^s_t(j)$ in period t; however, all firms use an identical technology, represented by the following production function:

$$C_t^s(j)=A_t\left(N_t^d(j)\right)^{1-\alpha}, \tag{66}$$

where we presuppose the absence of any capital accumulation in the firm[18]. A_t denotes the level of technology in period t, assumed to evolve exogenously over time as follows:

$$a_t=\rho_a a_{t-1}+\varepsilon_{a,t}, \tag{67}$$

where $a_t\equiv\ln(A_t)$, $\rho_a\in(0,1)$ and $\varepsilon_{a,t}\sim i.i.d.(0,\sigma_a^2)$, which indicates a shock to the technology. $N_t^d(j)$ stands for the number of work-hours hired from end-user borrowers and passive investors by firm j in period t, and $\alpha\in[0,1]$ represents the share of capital in production.

It is assumed that all firms face an identical isoelastic demand schedule, in which they take the aggregate consumer price level P_t in period t and aggregate consumption index C_t^d in period t as given. The demand schedule is described as follows:

$$C_t^d(j)=\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon}C_t^d, \tag{68}$$

where $P_t(j)$ is the price of consumption goods j in period t, ε stands for constant price elasticity, $C_t^d=C_{1,t}+C_{2,t}$ and

$$P_t\equiv\left[\int_0^1 P_t(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}}. \tag{69}$$

Here, let us consider the Calvo [1983] model of staggered price setting with the following modification. In the period between price reoptimizations, firms mechanically adjust their prices according to some indexation rule, described as the 'lagged inflation indexation' by Christiano, Eichenbanm and Evans [2005]. Formally, a firm that has the opportunity to reoptimize its price in period t with probability $1-\eta$ sets an optimal price P_t^* in that period. In subsequent periods (i.e. until the firm has the opportunity to reoptimize prices again), its price is adjusted according to the following rules of partial indexation to past inflation:

$$P_{t+k|t}=P_{t+k-1|t}(\Pi_{t+k-1})^{\omega} \tag{70}$$

for $k=1,2,\ldots$, and

$$P_{t,t}=P_t^*, \tag{71}$$

where $P_{t+k|t}$ denotes the price effective in period $t+k$ for the firm that last reoptimized its price in period t, and $\omega\in[0,1]$ is a parameter measuring the degree of indexation.

Combining the definition of aggregate consumer price (69) with the firm's price-adjusting rules (70) and (71), the aggregate consumer price dynamics are described as

$$\Pi_t^{1-\varepsilon}=\eta(\Pi_{t-1}^{\omega})^{1-\varepsilon}+(1-\eta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} \tag{72}$$

Next, we will derive a firm's optimal price setting. A firm reoptimizing in period t will choose the price P_t^* that maximizes the current market value of the profits generated, subject to a sequence of demand constraints and the rule of price adjustment. Based on the first-order conditions pertaining to the firm's optimizing problem, we can derive the following optimal price-setting equation:

$$\hat{p}_t^* - \hat{p}_t = (1-\beta\eta)\sum_{k=0}^{\infty}(\beta\eta)^k E_t\left[\widehat{mc}_{t+k|t} + (\hat{p}_{t+k} - \hat{p}_t) - \omega(\hat{p}_{t+k-1} - \hat{p}_{t-1})\right], \tag{73}$$

where $\beta \in (0, 1)$ is a discount factor of the hypothetical aggregated consumers, as defined below, and

$\widehat{mc}_{t+k|t} = \ln(MC_{t+k|t}) - \ln(\overline{MC})$

where $MC_{t+k|t}$ represents the real marginal cost in period $t+k$ for a firm whose price was last set in period t.

Finally, we will derive the NKPC. By combining the firm's production function (66), demand function (68) and market-clearing condition on consumption goods, we can derive the following approximate aggregated production function:

$$c_t = a_t + (1-\alpha)n_t^d, \tag{74}$$

where $n_t^d \equiv \ln(N_t^d)$ and $N_t^d = \int_0^1 N_t^d(j)dj$. Based on equation (74), an individual firm's marginal cost in terms of the economy's average real marginal cost can be defined as

$$mc_t = w_t - p_t - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \ln(1-\alpha). \tag{75}$$

Using equations (73) and (75), we can derive the following NKPC:

$$\hat{\pi}_t = \frac{\omega}{1+\omega\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\omega\beta}E_t\hat{\pi}_{t+1} + \frac{(1-\eta)(1-\beta\eta)}{\eta(1+\omega\beta)}\Theta\widehat{mc}_t \tag{76}$$

where $\Theta \equiv \dfrac{1-\alpha}{1-\alpha+\alpha\varepsilon} \leq 1$ and

$$\widehat{mc}_t = -\hat{\lambda}_{6,t} + \frac{\varphi+\alpha}{1-\alpha}\hat{c}_t - \frac{1+\varphi}{1-\alpha}\hat{a}_t. \tag{77}$$

Here, assuming that $h_1 \approx h_2$ and $\sigma_1 \approx \sigma_2$, we can approximately express β, φ and $\hat{\lambda}_{6,t}$ as follows:

$$\beta \approx \frac{\overline{C}_1}{\overline{C}}\beta_1 + \frac{\overline{C}_2}{\overline{C}}\beta_2, \tag{78}$$

$$\varphi = \left(\frac{\overline{N}^s}{\overline{N}_1^s}\right)\left(\frac{\overline{C}_1}{\overline{C}}\right)\varphi_1\left(=\left(\frac{\overline{N}^s}{\overline{N}_2^s}\right)\left(\frac{\overline{C}_2}{\overline{C}}\right)\varphi_2\right), \tag{79}$$

$$\hat{\lambda}_{6,t} \approx \frac{\overline{C}_1}{\overline{C}}\hat{\lambda}_{1,t} + \frac{\overline{C}_2}{\overline{C}}\hat{\lambda}_{5,t}. \tag{80}$$

For the derivation of equations (78) to (80), see Appendix.

2. The central bank

We assume that the central bank sets the one-period nominal (risk-free) interest rate as a policy rate based on an augmented Taylor-type interest rate rule. The interest rate in period t, R_t, responds not only to the rate in the previous period, and to deviations of the output and inflation rate from their steady-state values, but also to the ratio of the demand for money to the sum of base money and capital injection to the active investor with financial distress. Formally,

$$\ln\left(\frac{R_t}{\overline{R}}\right)=\rho_r\ln\left(\frac{R_{t-1}}{\overline{R}}\right)+(1-\rho_r)\left\{\rho_\pi\ln\left(\frac{\Pi_t}{\overline{\Pi}}\right)+\rho_y\ln\left(\frac{Y_t}{\overline{Y}}\right)+\rho_\mu\ln\left(\frac{\Phi_t}{\overline{\Phi}}\right)\right\}+e_{R,t}, \tag{81}$$

where $\rho_r\in(0, 1)$, $\rho_\pi>1$ (for fulfilling the Taylor principle), $\rho_y>0$ and $\rho_\mu>0$. Y_t denotes the real GDP in period t and is defined as follows[19]:

$$Y_t=C_{1,t}+C_{2,t}+(1-X_t)\{S_{h,t}-(1-\iota_h)S_{h,t-1}\}Q'_{h,t}. \tag{82}$$

$e_{R,t}$ is a shock to monetary policy and is expressed as

$$e_{R,t}=\rho_R e_{R,t-1}+\varepsilon_{R,t}, \tag{83}$$

where $\rho_R\in(0, 1)$, and $\varepsilon_{R,t}\sim i.i.d.(0, \sigma_R^2)$. In addition, Φ_t is defined as

$$\Phi_t\equiv\frac{M'_t}{B'_{r,t}+X_tL'_t}, \tag{84}$$

where L'_t is derived from equation (27).

3. Complete model

To close the model, we need to specify the market-clearing conditions on consumption goods, labour, and short- and long-term bonds. These conditions are expressed as follows:

$$C_t=C_{1,t}+C_{2,t}, \tag{85}$$

$$N_t^d=N_{1,t}^s+N_{2,t}^s, \tag{86}$$

$$B_{s,t}^{s'}=B_{s,t}^{d'}, \tag{87}$$

$$B_{l,t}^{s'}=B_{1,l,t}^{d'}+B_{2,l,t}^{d'}. \tag{88}$$

In addition, we must decide the processes of house prices $Q_{h,t}$ and the probability of default X_t. First, let us assume that the real house prices in period t depend on previous ones and housing stock, which is expressed by

$$Q'_{h,t}=GQ'^{\rho_{qh1}}_{h,t-1}S^{\rho_{qh2}}_{h,t-1}e_{qh,t}, \tag{89}$$

where $G>0$, $\rho_{qh1}\in(0, 1)$ and $\rho_{qh2}\in(0, 1)$ are parameters, and ρ_{qh1} and ρ_{qh2} should be $\rho_{qh1}+\rho_{qh2}<1$. $e_{qh,t}$ denotes a shock to the real house prices and is expressed as

$$e'_{qh,t}=\varepsilon_{qh,t}, \tag{90}$$

where $e'_{qh,t}\equiv\ln(e_{qh,t})$ and $\varepsilon_{qh,t}\sim i.i.d.(0, \sigma_{qh}^2)$.

Second, in order to link the process of $\{X_t\}$ to the business cycle, we presuppose that

$$X_t = \overline{X}(\overline{Y}/Y_{t-1})^{\rho_x}, \tag{91}$$

where $\overline{X} \in [0, 1]$ represents a steady state of the sequence $\{X_t\}$, and $\rho_x > 0$ is a parameter.

Ⅶ Conclusion

In this study, we developed a new-Keynesian DSGE model in which market-based intermediaries or active investors have an interactive relationship with the ultimate borrowers and lenders.

We could derive four propositions from the theoretical analysis. First, the collateral constraint on end-user borrowers is likely to push down housing stock in a steady state. This implies that the constraint controls residential investment. The second proposition is that, under a probable condition on active investors' investment, the marginal increases in capital would raise the expected net profits. This proposition indicates that, when an increase in the active investor's asset size induces a rise in capital, it pushes up the expected net profits. The third proposition states that the active investor's holdings of risky assets are affected by the slope of the yield curve; hence, the steeper the yield curve, the larger is its asset size. This proposition, together with the second one, suggests that a steeper yield curve is likely to yield larger net profits for active investors, which theoretically supports the views of Adrian, Moench and Shin [2010b], Cúrdia and Woodford [2010] and Woodford [2010]. Finally, the fourth proposition posits that the term structure of interest rates is endogenously shifted by the modified relative amounts of money and each bond that is outstanding. This proposition is consistent with Tobin's [1969] view.

Next step is to achieve the second objective stayed in introduction, based on the developed model: to empirically examine the propositions derived in this study, and to assess effects of the presence of the market-based intermediary sector on the business cycle. We will present the empirical study in the near future.

Appendix: Derivation of $\widehat{\lambda}_{6,t}$, β and ϕ

Let us consider the agents that comprise the end-user borrowers and passive investors. We assume that they have the following period utility function in stable times:

$$U_p(C_t, C_{t-1}, N_t^s, S_{h,t}, e_{h,t})=\frac{1}{1-\sigma_p}\left(\frac{C_t}{C_{t-1}^h}\right)^{1-\sigma_p}+\frac{S_{h,t}^{1-\chi}}{1-\chi}e_{h,t}-\frac{(N_t^s)^{1+\varphi_p}}{1+\varphi_p}, \tag{A.1}$$

and the following period utility function in times of financial distress:

$$U_d(C_t, C_{t-1}, N_t^s)=\frac{1}{1-\sigma_d}\left(\frac{C_t}{C_{t-1}^h}\right)^{1-\sigma_d}-\frac{(N_t^s)^{1+\varphi_d}}{1+\varphi_d}, \tag{A.2}$$

where $C_t=C_{1,t}+C_{2,t}$, $N_t^s=N_{1,t}^s+N_{2,t}^s$ and all parameters are defined in the same manner as in the main body of the text.

Following this, we can derive the constraints on the agents' behaviour from the setup illustrated in the main text. Hence, in stable times, the constraints are

$$\frac{1}{P_t}\left(W_tN_t^s+Q_{h,t}S_{h,t-1}+B_{l,t}^{*s}+H_{s,t-1}B_{s,t-1}^d+R_{t-1}M_{t-1}+T_t\right) \tag{A.3}$$

$$=C_t+\frac{1}{P_t}\left\{Q_{h,t}S_{h,t}+H_{l,t-1}B_{l,t-1}^{*s}+B_{s,t}^d+M_t(1+AC_{m,t})\right\},$$

and

$$\frac{1}{P_t}R_tB_{l,t}^{*s}\leq\frac{1}{P_t}k_hQ_{h,t}S_{h,t}, \tag{A.4}$$

where $B_{l,t}^{*s}\equiv B_{l,t}^s-B_{2,l,t}^d$. On the other hand, in financial distress, the constraint in period t is given by

$$\frac{1}{P_t}\left(W_tN_t^s+k_{bs}\gamma B_{l,t-1}^{*s}+R_{t-1}M_{t-1}+T_t\right)=C_t+\frac{1}{P_t}M_t, \tag{A.5}$$

where it is assumed that $B_{l,t}^{*s}$ is equal to $B_{l,t}^{*d}$, denoting long-term bond holdings by active investors. In period $t+i$ $(i\geq1)$, this equation becomes

$$\frac{1}{P_{t+i}}\left(W_{t+i}N_{t+i}^s+R_{t+i-1}M_{t+i-1}+T_{t+i}\right)=C_{t+i}+\frac{1}{P_{t+i}}M_{t+i}. \tag{A.6}$$

Based on the above setup, the agent solves the optimizing problem. Supposing $\sigma_p=\sigma_d=\sigma$ and $\varphi_p=\varphi_d=\varphi$, the first-order conditions on the optimizing problem suggest that

$$\Lambda_{6,t}=\frac{C_t^{-\sigma}}{C_{t-1}^{h(1-\sigma)}}-\beta hE_t\left[\frac{C_{t+1}^{1-\sigma}}{C_t^{h(1-\sigma)+1}}\right] \tag{A.7}$$

and

$$\left(N_t^s\right)^{\varphi}=\Lambda_{6,t}W_t', \tag{A.8}$$

where $\beta \in (0, 1)$ is a discount factor related to the agent's utility, and $\Lambda_{6,t}$ represents the Lagrange multiplier for the budget constraint.

1. The derivation of $\hat{\lambda}_{6,t}$

In a steady state, equations (11), (55) and (A.7) become, respectively,

$$\bar{\Lambda}_1=(1-\beta_1 h_1)\bar{C}_1^{\{(1-h_1)(1-\sigma_1)-1\}}, \tag{A.9}$$

$$\bar{\Lambda}_5=(1-\beta_2 h_2)\bar{C}_2^{\{(1-h_2)(1-\sigma_2)-1\}}, \tag{A.10}$$

$$\bar{\Lambda}_6=(1-\beta h)\bar{C}^{\{(1-h)(1-\sigma)-1\}}. \tag{A.11}$$

Accordingly, around the steady state, it can be presupposed that

$$\hat{\lambda}_{1,t}\approx\{(1-h_1)(1-\sigma_1)-1\}\hat{c}_{1,t}, \tag{A.12}$$

$$\hat{\lambda}_{5,t}\approx\{(1-h_2)(1-\sigma_2)-1\}\hat{c}_{2,t}, \tag{A.13}$$

$$\hat{\lambda}_{6,t}\approx\{(1-h)(1-\sigma)-1\}\hat{c}_{t}, \tag{A.14}$$

Substituting equations (A. 12) to (A. 14) into the market-clearing condition on consumption goods,

$$\frac{\bar{C}}{(1-h)(1-\sigma)-1}\hat{\lambda}_{6,t}\approx\frac{\bar{C}_1}{(1-h_1)(1-\sigma_1)-1}\hat{\lambda}_{1,t}+\frac{\bar{C}_2}{(1-h_2)(1-\sigma_2)-1}\hat{\lambda}_{5,t}. \tag{A.15}$$

Assuming that

$$(1-h)(1-\sigma)=(1-h_1)(1-\sigma_1)=(1-h_2)(1-\sigma_2), \tag{A.16}$$

equation (A.15) becomes

$$\hat{\lambda}_{6,t}\approx\frac{\bar{C}_1}{\bar{C}}\hat{\lambda}_{1,t}+\frac{\bar{C}_2}{\bar{C}}\hat{\lambda}_{5,t}. \tag{A.17}$$

2. The derivation of β

Equations (A.9) to (A.11) lead to the following equations:

$$\bar{\lambda}_1-\{(1-h_1)(1-\sigma_1)-1\}\bar{c}_1=\ln(1-\beta_1 h_1), \tag{A.18}$$

$$\bar{\lambda}_5-\{(1-h_2)(1-\sigma_2)-1\}\bar{c}_2=\ln(1-\beta_2 h_2), \tag{A.19}$$

$$\bar{\lambda}_6-\{(1-h)(1-\sigma)-1\}\bar{c}=\ln(1-\beta h), \tag{A.20}$$

Calculating $\left(\frac{\bar{C}_1}{\bar{C}}\right)\times$ (A.18) $+\left(\frac{\bar{C}_2}{\bar{C}}\right)\times$ (A.19) − (A.20) for both sides of these equations under the assumption in (A.16), and using the relationship applied to $\hat{\lambda}_1$, $\hat{\lambda}_5$ and $\hat{\lambda}_6$ in (A.17), we obtain

$$\ln(1-\beta h)\approx\left(\frac{\bar{C}_1}{\bar{C}}\right)\ln(1-\beta_1 h_1)+\left(\frac{\bar{C}_2}{\bar{C}}\right)\ln(1-\beta_2 h_2). \tag{A.21}$$

Presuming not only that βh, $\beta_1 h_1$ and $\beta_2 h_2$ are all small but also that h_1 and h_2 are around h, equation (A.21) indicates that

$$\beta\approx\left(\frac{\bar{C}_1}{\bar{C}}\right)\beta_1+\left(\frac{\bar{C}_2}{\bar{C}}\right)\beta_2. \tag{A.22}$$

3. The derivation of ϕ

Log-linearizing equations (12), (56) and (A. 8) around the steady state, we obtain the following equations, respectively:

$$\varphi_1\hat{n}^s_{1,t}=\hat{\lambda}_{1,t}+\hat{w}_t, \tag{A.23}$$

$$\varphi_2\hat{n}^s_{2,t}=\hat{\lambda}_{5,t}+\hat{w}_t, \tag{A.24}$$

$$\varphi\hat{n}^s_t=\hat{\lambda}_{6,t}+\hat{w}_t. \tag{A.25}$$

Next, calculating $\left(\frac{\bar{C}_1}{\bar{C}}\right)\times$ (A.23) $+\left(\frac{\bar{C}_2}{\bar{C}}\right)\times$ (A.24) − (A.25) and using the relationship applied to $\bar{\lambda}_1$, $\bar{\lambda}_5$ and $\bar{\lambda}_6$ in (A.17), we obtain

$$\varphi\hat{n}^s_t=\left(\frac{\bar{C}_1}{\bar{C}}\right)\varphi_1\hat{n}^s_{1,t}+\left(\frac{\bar{C}_2}{\bar{C}}\right)\varphi_2\hat{n}^s_{2,t}. \tag{A.26}$$

On the other hand, since $N^s_t=N^s_{1,t}+N^s_{2,t}$,

$$\hat{n}^s_t=\frac{\bar{N}^s_1}{\bar{N}^s}\hat{n}^s_{1,t}+\frac{\bar{N}^s_2}{\bar{N}^s}\hat{n}^s_{2,t}. \tag{A.27}$$

Substituting equation (A.27) into equation (A.26) and rearranging the resultant equation, we get

$$\left(\frac{\bar{N}^s_1}{\bar{N}^s}\varphi-\frac{\bar{C}_1}{\bar{C}}\varphi_1\right)\hat{n}^s_{1,t}+\left(\frac{\bar{N}^s_2}{\bar{N}^s}\varphi-\frac{\bar{C}_2}{\bar{C}}\varphi_2\right)\hat{n}^s_{2,t}=0. \tag{A.28}$$

In order to maintain the relationship in (A.28) for all t,

$$\frac{\bar{N}^s_1}{\bar{N}^s}\varphi=\frac{\bar{C}_1}{\bar{C}}\varphi_1 \text{ and } \frac{\bar{N}^s_2}{\bar{N}^s}\varphi=\frac{\bar{C}_2}{\bar{C}}\varphi_2.$$

As a result,

$$\varphi=\left(\frac{\bar{N}^s}{\bar{N}^s_1}\right)\left(\frac{\bar{C}_1}{\bar{C}}\right)\varphi_1\left(=\left(\frac{\bar{N}^s}{\bar{N}^s_2}\right)\left(\frac{\bar{C}_2}{\bar{C}}\right)\varphi_2\right). \tag{A.29}$$

Notes

1) Works from the theoretical viewpoint include, for example, Danielsson, Shin and Zigrand [2009] and Adrian and Shin [2011b]; those from the empirical viewpoint include, for example, Brunnermeier [2009], Adrian, Moench and Shin [2010a], and Adrian and Shin [2010b].
2) Adrian and Shin [2011a] explain these concepts in detail.
3) In this study, it is supposed that the shadow banks are composed of asset-backed security (ABS) issuers, finance companies, funding companies, and agency- and GSE-backed mortgage pools, where 'GSE' is an abbreviation for 'government-sponsored enterprises'. For a comprehensive and up-to-date description of the shadow banking system, see Pozsar et al. [2010].
4) It is implicitly assumed that borrowers not only demand houses but also supply them. Furthermore, we assume the presence of some suppliers other than borrowers, for example, homeowners by inheritance.
5) For example, the quarterly data on 10-year zero-coupon yields by the Federal Reserve System (FRB) indicate that $\overline{H}_l$ was around 1.014 during 1990:Q1 and 2010:Q3. Meanwhile, Iacoviello and Neri [2010] suggest that the impatient household's discount factor, β_1, is 0.97 for the U.S. Therefore, it could be said that the data pertaining to the U.S. bear out the condition of $1-\beta_1\overline{H}_l>0$.
6) When there is no collateral constraint, the borrower's discount factor, β_1^*, should be equal to $\overline{H}_l^{-1}$.
7) For example, using the federal funds rate and the 10-year zero-coupon yield for the U.S., $\overline{R}/\overline{H}_l$ is calculated as around 1.0018 from 1990:Q1 to 2010:Q3. Meanwhile, Iacoviello and Neri [2010] suggest that the values of ι_h and k_h are 0.01 and 0.925, respectively. These data and parameter values support the condition that $k_h<(1-\iota_h)\overline{R}/\overline{H}_l$.
8) This implies that at the beginning of period t, active investors repurchase short-term bonds issued in a previous period at a price $Q_{s,t}$, while they simultaneously issue the bond at the same price.
9) Equation (23) and definition (24) imply that in a steady state, $\overline{R}\,\overline{B}_r+\overline{H}_l\overline{B}_{1,l}^d-\overline{R}\,\overline{M}-\overline{H}_s\overline{B}_s^s=E_m$. From the viewpoint of an optimal capital-to-assets ratio, ϕ, the capital adequacy requirement is expressed as $\frac{E_m}{\overline{R}\,\overline{B}_r+\overline{H}_l\overline{B}_{1,l}^d}=\phi$, where, for example, $\phi=0.08$ according to the Basel Accords.
10) From the viewpoint of a value-at-risk (VaR) constraint, as suggested by Adrian and Shin [2011a], we should incorporate $\gamma H_{l,t}B_{1,l,t}^d$ instead of $H_{l,t}B_{1,l,t}^d$ into the budget constraint (23), because if end-user borrowers face financial distress, they can repay $\gamma H_{l,t}B_{1,l,t}^d$ to active investors. However, since our model explicitly considers the probability of default, it adopts a normal budget constraint in stable times.
11) The parameter $v_{1,l}$ is conceptually similar to κ_{kb} in Gerali et al. [2010], which indicates an adjustment coefficient on the quadratic cost that the bank pays whenever the capital-to-assets ratio moves away from the optimal value (in our context, ϕ).
12) In this study, since we do not consider a capital increase for the active investor, we can regard the liquidity requirement as playing the role of a kind of capital adequacy regulation.
13) For example, using the 3-month TB rate and the federal funds rate for the U.S., $\overline{R}/\overline{H}_s=1.0006$ from 1990:Q1 to 2010:Q3. Consequently, the collateral constraint is binding around a steady state for any $\overline{X}\in[0,1]$.
14) This study defines the slope of the yield curve in terms of holding returns instead of interest rates. The validity of this definition is given in Appendix A of Sudo [2010].
15) Adrian, Moench and Shin [2010b] state that when the financial intermediary's balance sheet constraints are loose, the risk premia are compressed. Their argument seems to contradict proposition 3. This is because they relate the intermediary's balance sheet size to the time difference in the weighted combination of yield and credit spreads (i.e. r_t-r_{t-1} in their context), whereas we associate it with the term premium of holding returns (i.e. $r_{t+1}-r_t$ in their context). Proposition 3 is consistent with Woodford's [2010] view that a larger credit spread encourages financial intermediaries to increase the supply of credit.
16) The cost function has another implication in that this functional form of $AC_{m,t}$ guarantees non-zero demand for these riskier assets, in terms of the passive investor budget constraint, under the condition that all $v_{2,s}$, $v_{2,l}$, $\kappa_{2,s}$ and $\kappa_{2,l}$ are positive.
17) For further details, see, for instance, Sudo [2010].
18) In equation (66), $C_t^s(j)$ and $N_t^d(j)$ are interpreted as per capital in period t for firm j.
19) According to the definition of Y_t, the monetary policy implicitly takes house prices (i.e. asset prices) into consideration.

References

Adrian, T. and H. S. Shin [2010a] "The Changing Nature of Financial Intermediation and the Financial Crisis of 2007-09," *Annual Review of Economics,* Vol.2, pp.603-618.

Adrian, T. and H. S. Shin [2010b] "Liquidity and Leverage," *Journal of Financial Intermediation,* Vol.19 (3), pp.418-437.

Adrian, T. and H. S. Shin [2011a] "Financial Intermediaries and Monetary Economics," in B. M. Friedman and M. Woodford (eds.), *Handbook of Monetary Economics,* Vol.3, Chapt.12, Amsterdam: Elsevier.

Adrian, T. and H. S. Shin [2011b] "Procyclical Leverage and Value-at-Risk," *Federal Reserve Bank of New York Staff Reports,* No.338 (Revised version of the Staff Report no.338 published in July 2008).

Adrian, T., E. Moench and H. S. Shin [2010a] "Financial Intermediation, Asset Prices and Macroeconomic Dynamics," *Federal Reserve Bank of New York Staff Reports,* No.422.

Adrian, T., E. Moench and H. S. Shin [2010b] "Macro Risk Premium and Intermediary Balance Sheet Quantities," *Federal Reserve Bank of New York Staff Reports,* No.428.

Aliaga-Diaz, R. and M. P. Olivero [2007] "Macroeconomic Implications of Market Power in Banking," *Working Paper,* LeBow College of Business, Drexel University.

Andrés, J. and O. Arce [2009] "Banking Competition, Housing Prices and Macroeconomic Stability," *Working Paper,* No.0830, Banco de Espana.

Andrés, J., J. D. López-Salido and E. Nelson [2004a] "Tobin's Imperfect Asset Substitution in Optimizing General Equilibrium," *Journal of Money, Credit, and Banking,* Vol. 36 (4), pp. 665-690.

Andrés, J., J. D. López-Salido and E. Nelson [2004b] "Money and the Natural Rate of Interest: Structural Estimates for the UK, the US and the Euro Area," *CEPR Discussion Papers,* No. 4337.

Bernanke, B. S., M. Gertler and S. Gilchrist [1999] "The Financial Accelerator in a Quantitative Business Cycle Framework," in J. B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics,* Vol.1, Chapt. 21, Amsterdam: Elsevier.

Brunnermeier, M. [2009] "Deciphering the Liquidity and Credit Crunch 2007-2008," *Journal of Economic Perspectives,* Vol.23 (1), pp.77-100.

Calvo, G. [1983] "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics,* Vol. 12 (3), pp.383-398.

Christiano, L. J., M. Eichenbaum and C. L. Evans [2005] "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy,* Vol.113 (1), pp.1-45.

Christiano, L. J., R. Motto and M. Rostagno [2003] "The Great Depression and the Friedman-Schwartz Hypothesis," *Journal of Money, Credit, and Banking,* Vol.35 (6, Part 2), pp.1119-1197.

Christiano, L. J., R. Motto and M. Rostagno [2008] "Shocks, Structures or Monetary Policies? The Euro Area and US after 2001," *Journal of Economic Dynamics and Control,* Vol.32 (8), pp.2476-2506.

Christiano, L. J., R. Motto and M. Rostagno [2010] "Financial Factors in Economic Fluctuations," *Working Paper Series,* No.1192, European Central Bank.

Cúrdia, V. and M. Woodford [2010] "Credit Spreads and Monetary Policy," *Journal of Money, Credit, and Banking,* Vol. 42 (Supp.1), pp.3-35.

Danielsson, J., H. S. Shin and J-P. Zigrand [2009] "Risk Appetite and Endogenous Risk," *Working Paper,* London School of Economics. (http://www2.lse.ac.uk/fmg/workingPapers/discussionPapers/Risk%20 Appetite%20and%20Endogenous%20Risk.pdf)

Gerali, A., S. Neri, L. Sessa and F. M. Signoretti [2010] "Credit and Banking in a DSGE Model of the Euro

Area," *Journal of Money, Credit, and Banking,* Vol.42 (Supp.1), pp.107-141.

Gertler, M. and N. Kiyotaki [2011] "Financial Intermediation and Credit Policy in Business Cycle Analysis," in B. M. Friedman and M. Woodford (eds.), *Handbook of Monetary Economics,* Vol.3, Chapt.11, Amsterdam: Elsevier.

Gilchrist, S, A. Ortiz and E. Zakrajsek [2009] "Credit Risk and the Macroeconomy: Evidence from an Estimated DSGE Model," paper prepared for the FRB/JMCB conference "Financial Markets and Monetary Policy," held at the Federal Reserve Board, Washington D. C., June 4-5, 2009. (http://sws.bu.edu/aortizb/Credit%20Risk%20and%20the%20Macroeconomy%20Gilchrist%20Ortiz%20%20Zakrajsek.pdf)

Goodfriend, M. and B. T. McCallum [2007] "Banking and Interest Rates in Monetary Policy Analysis: A Quantitative Exploration," *Journal of Monetary Economics,* Vol.54 (5), pp.1480-1507.

Iacoviello, M. and S. Neri [2010] "Housing Market Spillovers: Evidence from an Estimated DSGE Model," *American Economic Journal: Macroeconomics,* Vol.2 (2), pp.125-164.

Kiyotaki, N. and J. Moore [1997] "Credit Cycles," *Journal of Political Economy,* Vol.105 (2), pp.211-248.

Kobayashi, T. [2008] "Incomplete Interest Rate Pass-Through and Optimal Monetary Policy," *International Journal of Central Banking,* Vol.4 (3), pp.77-118.

Marzo, M., U. Söderström and P. Zagaglia [2007] "The Term Structure of Interest Rates and the Monetary Transmission Mechanism," (http://bankoffinland.fi/NR/rdonlyres/D4CB5764-C768-48DD-9DCE-82CB8C3DCAA4/0/Zagaglia_Feb2008.pdf).

Pintus, P. A. and Y. Wen [2008] "Excessive Demand and Boom-Bust Cycles," *Working Paper Series,* 2008-014B, Federal Reserve Bank of St. Louis.

Pozsar, Z., T. Adrian, A. Ashcraft and H. Boesky [2010] "Shadow Banking," *Federal Reserve Bank of New York Staff Reports,* No.458.

Sudo, T. [2010] "Effects of Monetary and Fiscal Policy on the Yield Curve: Evidence from the U.S. and U.K.," *JSRI Journal of Financial and Securities Markets* (*Shoken Keizai Kenkyu*), No.71, pp.1-37.

Teranishi, Y. [2008] "Optimal Monetary Policy under Staggered Loan Contracts," *IMES Discussion Paper Series,* 2008-E-08, Institute for Monetary and Economic Studies, Bank of Japan.

Tobin, J. [1969] "A General Equilibrium Approach to Monetary Theory," *Journal of Money, Credit, and Banking,* Vol. 1 (1), pp. 15-29.

Woodford, M. [2010] "Financial Intermediation and Macroeconomic Analysis," *Journal of Economic Perspectives,* Vol.24 (4), pp.21-44.

Verona, F., M. M. F. Martins and I. Drumond [2011] "Monetary Policy Shocks in a DSGE Model with a Shadow Banking System," *CEF. UP Working Paper,* 2011-01, Centro de Economia e Financas de UP.